The optimal spatial distribution of small and large scale fisheries

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Abstract

In this paper the problem of spatially sharing a common fish resource between two quite different harvesting activities is modelled. The problem is studied from the viewpoint a of regulator in charge of deciding how to share the resource. We assume that the regulator has to fix the percent of the area opened to each of the agents and in this way design two patches. One of the patches should be used only by small scale fisheries and plays the role of a reserve that contains the reproduction area. In the second patch a more efficient and intensive activity should be permitted. This problem is motivated by a study case in Chile. The model is introduce stepwise and takes into account the different impact of the activities and a natural biomass transfer into the patches. We end up with an optimal control problem that try to find social relevant solutions. The resulting control problem is treated using a calculus of variation approach and some conditions ensuring the existence of interior solutions are proposed. Finally some academical tests are done in order to see the feasibility of the proposed conditions.

1 Introduction

The International Conference on the Economics of Marine Protected Areas (MPA) held in July 2000 in Vancouver was one of the starting points in the development of a new paradigm: the use of MPA as an instrument for the management of fisheries. Since then, different applications of these concepts have been developed to analyze a wide number of conflicts currently presents in fisheries all around the world, see for instance [20, 21]. The increasing number of studies on this area also shows a growing interest for the use of MPA as a valid instrument for fisheries management [5, 7, 10, 11, 16, 20, 21].

The use of marine reserve as a fishery management tool is controversial, see [2, 8, 10, 11, 13, 22]. This is understandable, since this tool combines conservation benefits with commercial gain.

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Sanchirico and Wilen [23] seem to be first on suggesting in the economic literature the possibility that MPA could be beneficial not only from an ecological but also from a economic point of view. In their dynamic and spatial model of a Marine Reserve Creation they analyze the idea that the transfer of biomass from the reserve to areas where catch is allowed could create economics profit from the MPA, since its creation could help to improve a depleted biomass and increase catch outside of the reserve. They call this a *double-payoff* because in this case the MPA will increase both the biomass and the economics profits from the fishery. They considered several possibilities for the biomass transfer mechanism between patches in their spatial model, and make use of differential equations for modelling biomass change and effort change between patches.

As noted by Sumaila and Charles [25], the research on the use of MPA has tend to focus in two areas. The first area is related to describe the potential ecological benefits, in terms of ecosystem health, biodiversity and greater long-term harvest, see for instance [14, 18, 19, 26]. The second area is devoted to optimize the process by which an MPA is developed and implemented, see for instance [11, 12, 23].

From the very first papers on the topic it was already clear that the design and the management of a reserve area are crucial in order to meet the biological or economical goals more effectively. Consequently, the design and management of MPA have been an active area of research in the last years, see for instance [2, 8, 9, 11, 15, 17, 24]

In this paper we develop an optimization model to analyze a fisheries management problem currently present in Chile. The problem consists on dividing a marine space where pelagic fish is found, between two users of the resource. The users are faced to quite different technologies, cost and regulatory systems to develop their activities. In our case, the users are, on the one hand, the small-scale fishermen who are allowed to develop their activities in places near the coast (let us define it as the small-scale exclusive area). On the other hand there are the large-scale fishermen who are not allowed to fish within the small-scale exclusive area near to the coast. There exist certain biomass transfer basically from the coast to the non exclusive are. However, the main part of the population is found inside the small-scale exclusive area. In this sense the large-scale users do not gain a significant benefit from the lower level of activity in the exclusive area.

In this scenario an spatial externality can be clearly identified, since the small-scale fishery does not have enough effort capacity to capture the biological surplus of the biomass inside the exclusive area. There is, therefore, periodically a political discussion whether (and to which scale) the industrial fishery should be authorized to enter to the area reserved to the small-scale fishery. Due to the difference in the effort capacities, and because this permission is spatial, allowing the industrial fishery inside the exclusive area would maintain the similarity of the situation with an MPA management tool. The most controversial design problem remains to fix the percent of the whole exclusive area that should be opened to industrial activity.

The general problem of identifying the optimal design of an MPA is considered in many recent papers in the literature. For instance, in the work of Pezzey, et.al. [17] the effect of a no-take reserve area on the catch of an outside area is studied. Using the bioeconomic equilibrium for biomass growth and a zero profit condition for the effort, they calculate the levels of the stock and catch for different sizes of a reserve. With this information the problem of finding the optimal size of the MPA is analyzed. However, they do not explicitly model the optimal choice problem appearing by setting simultaneously the optimal size of the reserve and effort level.

In more recent papers other authors also studied the double pay-off of a marine reserve using a model to find a social optimal reserve, see for instance Dubey et al. [9] or Ami et al. [2]. In the latter one the effort outside of the reserve and a parameter that controls biomass transfer between patches are considered into a dynamic optimization model. The authors use then a calculus of variation approach to look for an optimal steady state of the controlled system. We closely follow the methodology in [2] but are faced with a more difficult case.

The aim of this paper is then to use the concepts on MPA for developing a model to optimally design the exclusive access area for small-scale fishermen but allowing to certain percent the industrial activity. The model should consider both the lower impact of the small-scale fishermen and the biomass transfer mechanisms and allows the industrial fishery to actually benefit from a net increase of the biomass.

The paper is organized as follows. In the second section a detailed description of the problem that motivated this study is given. In the third section the dynamical system describing the biomass is discussed and its main properties presented. The different items involved in the biomass modelling, as harvesting and the shared growth region, are introduced separately.

The fourth section is devoted to the optimization problem. The calculus of variation approach is briefly explained and the main results concerning the Euler equations are given. Finally a short subsection presents a numerical example of the results. The pape end with a short section of concluding remarks, an appendix containing all the mathematical proofs.

2 The spatial distribution between fisheries.

The Chilean Act of Fishing and Acuiculture ("Ley General de Pesca y Acuicultura" in Spanish, [1]) recognize the existence of both small-scale fishermen ("pescadores artesanales") and large-scale fishermen ("pescadores industriales"). In the article 2nd.of the LGPA an operator of the large-scale fishery is defined as an individual registered on the industrial registry, which develops on his own behalf and risk a fishery extractive activity, using one or more fishing vessels whatever its type, size, design or specialization. In the same second article the small-scale fishermen are intended as the owners of up to two small-scale vessels, which together can not exceed 50 tons of gross registered tonnage (GRT). The LGPA also specify a small-scale vessel as one as such registered and having a maximum length of 18 meters and up to 50 tons GRT. Even when these definitions are quite general, there exist also more specific rules distinguishing who can be registered as an artisanal or as an industrial fishermen in the corresponding registry.

Additionally, the law establishes different regulatory mechanisms for small-scale and large-scale fishery. The most important distinction is given in the article 47 where an area of exclusive access to the small-scale fishermen is fixed. The area corresponds to the first five nautical miles from the normal base lines and covers around 6600 square nautical miles (Bernal, Oliva and Aliaga, [3]). The same article 47 of the LGPA also mentions that nevertheless, in some specific cases the regulatory agency might authorize the industrial fleet to access to a portion of the artisanal reserve area. This is the fact that generates continues petitions form the large-scale fishermen to access this area and which motivates the research that is the main goal of this paper.

Additionally both fisheries have different access rules and regulatory mechanisms. For example, while in the artisanal fishery the regulation is performed mainly through effort and technological control and temporarily closure of some areas, in the industrial fishery different regulatory schemes are applied. In this case global quotas and Individual Quotas (IQs) can also be used to manage the effort. We consider that this fact give more flexibility to fishermen to optimally adjust the effort, especially when IQs are used on the industrial fisheries.

From a biological point of view, it has been argued that the first five nautical miles have an important function as an area of primary and secondary reproduction, spawn and recruitment. This is increased because the phenomena of coastal upwelling. Upwelling occurs in this zone when southeasterly trade winds , produced by the South Pacific anti-cyclone, along with other factors drive coastal waters out to sea, forcing deep nutrient-rich waters to rise.

This reason motivate us to model the biomass transfer process as one of the Sink-Source type, where a net transfer of biomass is generated from the coast to the open see where the industrial fishery activity is developed.

3 The basic biomass model

To model the problem we are facing and to be able to find corresponding solutions we need to make some assumptions.

First we assume that the whole stock z obeys a logistic growth law.

$$\dot{z} = \gamma z(1 - \frac{z}{K}), \tag{1}$$

with K and γ denoting the total carrying capacity and instantaneous growth of the stock. Now we assume the existence of two oceanographically determined areas (near and far from the coast). In the area near to the coast occurs the reproduction. The stock does not growth in the other area, but it receives transfers of biomass from the first one. Let us then consider the whole stock $z = x_1 + x_2$ as the sum of the stock in the first x_1 and the second x_2 area. We assume that a net transfer of biomass exists from the growth to the other area.

Assuming that the rate of biomass transfer is given by a constant parameter b in $[0,1]$, we state the following model without harvesting:

$$\begin{array}{rcl} \dot{x}_1 & = & \gamma(x_1+x_2)\left(1-\frac{1}{K}(x_1+x_2)\right)-bx_1 \\ \dot{x}_2 & = & bx_1 \end{array} \quad (2)$$

Note that adding both equalities lead us to the original model (1). This second model (2) has the two equilibria $(0,0)$ and $(0,K)$, where only the second one is stable. It could be surely more realistic to assume a lower rate of growth in the second area, but we desisted this way in order to simplify the technical analysis.

3.1 Sharing the growth area

Now we have that the historical access rights are given for the artisanal fishermen in the first "growth-area". However we want to optimally determine the percent of this area that should be opened to the industrial fleet. We will call artisanal reserve to the area in which exclusive access rights will remain given to the artisanal sector. We model the design of the artisanal reserve through the parameter λ in [0,1] which represents the portion of the growth-area reserved for artisanal use. To be consistent with the previous literature we call this λ portion of the growth the artisanal patch. The remaining region, given by the area far from the coast and the resting $(1-\lambda)$ portion of the growth area, will be then called the industrial patch. We keep the notation, such that, the variables corresponding to the artisanal patch shall have the sub-index 1 and the variables corresponding to industrial patch the sub-index 2.

For modeling the biomass growth we assume that in each patch the natural growth is proportional to the size of the growth area assigned to each patch. The implicit assumption here is that the biomass is homogeneously distributed in the growth area. We stick assumming that the net transfer of biomass exists only from the artisanal to the industrial patch and this to a constant rate b in $[0,1]$. Taking into account the above assumptions we obtain the model still without harvesting:

$$\begin{array}{rcl} \dot{x}_1 & = & \lambda\gamma(x_1+x_2)\left(1-\frac{1}{K}(x_1+x_2)\right)-bx_1 \\ \dot{x}_2 & = & (1-\lambda)\gamma(x_1+x_2)\left(1-\frac{1}{K}(x_1+x_2)\right)+bx_1 \end{array} \quad (3)$$

In a more accurate model the rate of transfer would depend on the parameter λ and also on the geometry of the portion selected. This geometrical dependence would capture the spatial natural flow of the species inside the whole region and the specific way the region is subdivided. Due to this spatial-geometric consideration the rate of transfer can be different for equal values of the proportion λ. It can, in fact, for some particular geometries even be independent of λ, and equal, for instance, to zero. This analysis would lead to much more complex models, which are far beyond the purposes of this work. Fortunately, our assumption enormously simplify the tractability of the problem and allow us to find reasonable solutions without affecting the main conclusions of our analysis.

With this definitions, even if all the growth area is defined as artisanal reserve ($\lambda = 1$) and therefore the industrial patch will not have any natural growth, it still will have stock growth and fishing activity will be possible in a steady state because the transfer of biomass from the artisanal patch. If otherwise $\lambda < 1$ the stock under industrial fishing will present some natural growth corresponding to the fraction of the growth area that is in the industrial patch and also will present some transfer of biomass from the artisanal patch (see figure 1).

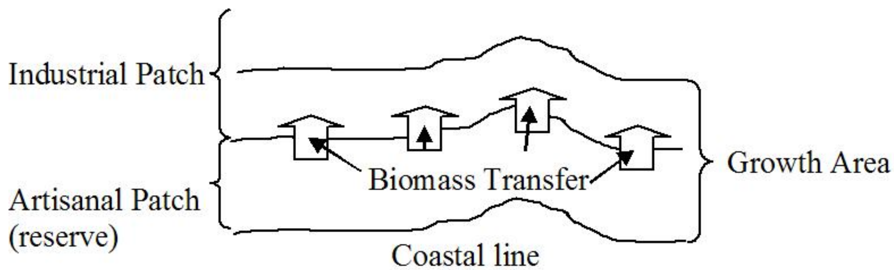

Figure 1: Ilustration of Growth-area, Industrial and Artisanal Patches

We want to underline that in our model the dynamics of the whole populations is recovered, when we add the two equations. This consistency is not present in many models found in the literature of patchy environments.

3.2 Including the harvesting

Let us now introduce harvesting into the model. As expected the net growth of biomass in each path would be then given by the natural growth plus/minus the net transfer of biomass between patches minus the harvest from fishing activity.

We take the catch functions as the product of a constant technological parameter (q_i), the efforts level (E_i) and the stock level (x_i) in each patch. Since the efforts level in the artisanal fishery is assumed to be very limited it shall be considered constant over time (denoted by E_1). The level of industrial effort should be optimally determined and shall be denoted by E_2. The biomass is then supposed to obeys the following dynamical system:

$$\begin{aligned} \dot{x}_1 &= \lambda\gamma(x_1+x_2)\left(1-\tfrac{1}{K}(x_1+x_2)\right) - bx_1 - q_1E_1x_1 \\ \dot{x}_2 &= (1-\lambda)\gamma(x_1+x_2)\left(1-\tfrac{1}{K}(x_1+x_2)\right) + bx_1 - q_2E_2x_2 \end{aligned} \qquad (4)$$

Here the parameters q, b, E_1 are positive and fixed. The variables $\lambda \in [0, 1]$ and $E_2(t)$ are to be determined later by the optimization modelling.

A basic assumption in the model is the following one

$$\gamma > q_1E_1 \qquad (5)$$

This inequality just ensures that the rate of harvesting in the artisanal patch (playing the role of the reserve!) does not exceed the instantaneous growth of the stock. This condition is in the above sense very natural and appears, for instance, in [6] as a necessary condition for the existence of a positive equilibria in the simple Schaeffer model.

We are interested in the study of the above system in the case that the control variables are fixed (and specially the effort E_2 constant). This analysis is specially important in order to consider interior solutions.

Let us therefore fix for a while $E_2(t) = \bar{E}_2$ and $\lambda = \bar{\lambda}$ as constant controls satisfying

$$\begin{array}{rcl} \bar{E}_2 & > & 0, \\ \bar{\lambda} & \in & [0,1]. \end{array} \tag{6}$$

The above are just feasibility conditions over the controls. A more technical assumption would be

$$\gamma > q_2 \bar{E}_2 \tag{7}$$

This assumption however has the same flavor and meaning of the above condition (5), i.e. in non of the patches the catch rate should exceed the natural growth rate of the biomass.

Under the above three conditions the system (4) has only two stationary points. One of them is the zero, which is not stable. The other one is strictly positive, stable and given by

$$\bar{x}_1 = \frac{K}{\gamma}\bar{\lambda} q_2 \bar{E}_2 \frac{(\gamma - q_2\bar{E}_2)(b + q_1 E_1) - \bar{\lambda}\gamma(q_1 E_1 - q_2 \bar{E}_2))}{[(b + q_1 E_1) - \bar{\lambda}(q_1 E_1 - q_2 \bar{E}_2)]^2} \tag{8}$$

$$\bar{x}_2 = \frac{\bar{x}_1}{\bar{\lambda} q_2 \bar{E}_2}(b + (1 - \bar{\lambda}) q_1 E_1) \tag{9}$$

A proof of the above facts can be found in the Appendix (see Proposition 1).

4 The optimization problem

We assume for the rest of the paper a constant price of fish, denoted by p, and constant costs of effort in both sectors given by c_i.

Our objective is then to maximize the current value of the profits from both artisanal and industrial fishery by choosing the size of the artisanal reserve (given by the parameter λ) and the industrial effort (E_2). This approach is captured by the following model.

$$\begin{array}{rl} \max\limits_{E_2(.),\lambda} & \int_0^{+\infty} e^{-\delta t}\left[\left(pq_2 x_2(t) - c_2\right) E_2(t) + \left(pq_1 x_1(t) - c_1\right) E_1\right] dt \\ s.t. & \begin{array}{rcl} \dot{x}_1 & = & \lambda\gamma(x_1 + x_2)\left(1 - \frac{1}{K}(x_1 + x_2)\right) - bx_1 - q_1 E_1 x_1 \\ \dot{x}_2 & = & (1-\lambda)\gamma(x_1 + x_2)\left(1 - \frac{1}{K}(x_1 + x_2)\right) + bx_1 - q_2 E_2 x_2 \end{array} \\ & 0 \le E_2(t); \lambda \in [0,1] \end{array} \tag{10}$$

where p, q, c, b, E_1 are the above introduced real and positive constant parameters.

This optimization problem can be reformulated using the Calculus of Variation Approach (see [2] for a similar construction). Adding up the two differential equations in (4) the variable λ disappears. If we further isolate the other control $E_2(t)$ variable and substitute it in the objective, the optimization model transforms into the following:

$$\max_{x_1(.),x_2(.)} \quad \int_0^{+\infty} e^{-\delta t} l(x_1(t), x_2(t), \dot{x}_1(t), \dot{x}_2(t))dt \tag{11}$$

where

$$l(x, \dot{x}) = (p - \frac{c_2}{q_2 x_2})[\gamma(x_1 + x_2)(1 - \frac{x_1 + x_2}{K}) - q_1 E_1 x_1 - \dot{x}_1 - \dot{x}_2] + (pq_1 x_1 - c_1)E_1$$

The theory of Calculus of variations is well established in the finite horizon case. However, the situation is completely different in our infinite horizon framework as (11). Nevertheless it is known that if the problem is considered over the BC^1 space (bounded curves defined on $[0, +\infty)$ with bounded derivatives), then the first order conditions of optimality given by the Euler-Lagrange equations holds true, see [4].

The Euler-Lagrange equations of the problem (11) are given by the following dynamical system (see Proposition 2 in the Appendix):

$$\begin{aligned} \dot{x}_1 &= x_2(\tfrac{pq_2}{c_2}x_2 - 1)[\gamma(1 - 2\tfrac{x_1+x_2}{K}) - \delta] + \gamma(x_1 + x_2)(1 - \tfrac{x_1+x_2}{K}) - q_1 E_1 x_1 \\ \dot{x}_2 &= -x_2(\tfrac{pq_2}{c_2}x_2 - 1)[\gamma(1 - 2\tfrac{x_1+x_2}{K}) - \delta] \end{aligned} \tag{12}$$

The optimality conditions also include some constraints concerning the feasibility of the controls E, λ, but we do not give them explicitely here. Since we are mainly interested in constant solutions of the problem we want to study the feasible stationary points of the above dynamics.

A fundamental step in our approach is to identify conditions ensuring that the above Euler-Lagrange systems has exactly one stationary point with strictly positive components. Under the assumption (5) we have different conditions ensuring that. One of them is defined as follows:

$$0 < \frac{q_1 E_1}{(\gamma + \delta - 2q_1 E_1)} < \left(\frac{p_2 q_2 K}{c_2} \frac{(\gamma - q_1 E_1)}{\gamma} - 1\right) \tag{13}$$

Another sufficient system of inequalities is the following one:

$$2q_1 E_1 < \gamma + \delta \tag{14}$$

$$\frac{c_2}{pq_2 K} < \left(1 - \frac{q_1 E_1}{\gamma + \delta - q_1 E_1}\right)\left(1 - \frac{q_1 E_1}{\gamma}\right) \tag{15}$$

Both conditions are sufficient for the existence of a unique strictly positive stationary point of the Euler-Lagrange system (see Proposition 3 of the Appendix). In both cases the unique stationary point $x^* = (x_1^*, x_2^*)$ is given by:

$$x_1^* = \frac{K}{\gamma}(\gamma - q_1 E_1) - \frac{c_2(\gamma + \delta - q_1 E_1)}{pq_2(\gamma + \delta - 2q_1 E_1)} \tag{16}$$

$$x_2^* = \frac{c_2(\gamma + \delta - q_1 E_1)}{pq_2(\gamma + \delta - 2q_1 E_1)} \tag{17}$$

The next step is to calculate which constant controls should be selected in order to actually realize for the original system (4) the above solution x^* as a stationary point. It is not difficult to prove (see Proposition 4 in the Appendix), that the right selection is to take

$$E_2^* = \frac{q_1 E_1}{q_2} \tag{18}$$

$$\lambda^* = \frac{q_1 E_1 + b}{q_1 E_1 \left(1 + \frac{x_2^*}{x_1^*}\right)} \tag{19}$$

and that both controls turn out to be positive.

The condition (18) suggest that the rate of harvesting should be identical on both patches. Note also that this conditions together with (5) implies straightforwardely the technical condition (7) assumed in the above section.

Finally the feasibility of the above calculated controls should be analyzed. It suffices in our case to ensure that $\lambda \in [0, 1]$. This is certainly true for sufficiently small values of b.

This implies that in some cases the steady state of both industrial and artisanal sectors can optimally subsist with positive levels of biomass and catches. Notice that in the optimal steady state the biomass level in each patch does not depends on the biomass transfer between patches. Moreover, even when there does not exist any transfer between patches, an optimal reserve exists with lambda in (0,1).

A simple condition for the feasibility of the calculated controls is the following

$$b < q_1 E_1 \frac{x_2^*}{x_1^*}. \tag{20}$$

This implicitly gives a lower bound for the value of the transfer parameter, for which protection of the complete growth-area as an artisanal reserve is justified. For lower values of the transfer parameter, it can be better to some extend to allow the industrial fleet to develop activities in the growth area.

4.1 Numerical example

In this short subsection we just want to test the feasibility of the model using academical values for the parameters. Our main interest is to see that the conditions stated theoretically are realizable. Another point to check is that the values obtained for the controls are feasible and leading to reasonable trajectories and objective values.

Let us fix then the parameters associated to the dynamical systems in the constraints as:

Biological quantities:		**Harvesting parameters:**	
Instantaneous growth	$\gamma = 0.16$	Artisanal capturability	$q_1 = 0.1$
Carrying capacity	$K = 100$	Industrial capturability	$q_2 = 1$
Biomass Transfer	$b = 0.2$	Artisanal effort	$E_1 = 1$

Finally the parameters to be used in the objective functions are the following ones:

Economical parameters:	
Price per unit harvest	$p = 100$
Cost per unit of artisanal effort	$c_1 = 50$
Cost per unit of industrial effort	$c_2 = 500$
Discount factor	$\delta = 0.06$

For the above values for the parameters the assumptions (5),(14) and (15) are satisfied. We obtain then the uniquely determined solution

$$\begin{aligned} x_1^* &= 7.5 \\ x_2^* &= 30 \end{aligned}$$

Moreover the condition (20) also holds true, giving rise to the following feasible constant controls.

$$\begin{aligned} E_2^* &= 0.1 \\ \lambda^* &= 0.6 \end{aligned}$$

Let us also note that the function involved in the objective function is also positive in the stationary point (i.e. $l(x_1^*, x_2^*, 0, 0) = 275 > 0$).

Finally, the following figure shows the trajectories of the dynamical system describing both populations for different start points.

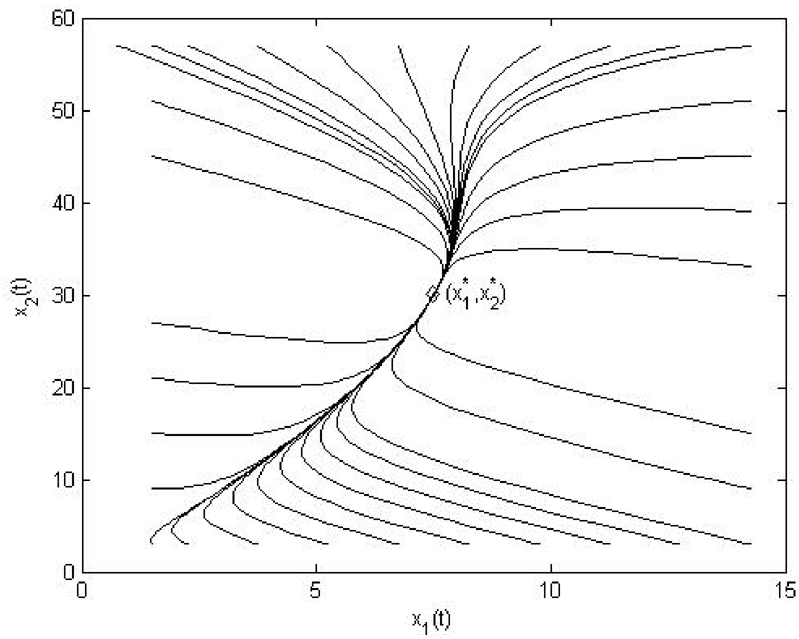

Figure 2: Trajectories of the system (4) using the controls (18-19)

5 Conclusions

We have considered a dynamical system for modelling a population distributed near to a coastal area. The area is supposed to be legally separated into two patches to be determined. It has been supposed that only one of the patches shall contain the main reproduction area. Consequently a natural transfer to the other patch can be assumed.

In our model the introduction of patches has been done preserving the natural properties of the whole population. In fact, adding both dynamical equations results into a logistic law for the whole population (the sum of both patches) and this, independently of how the patches were fixed.

The source patch plays the role of a quasi reserve since only artisanal activity shall be allowed there. The second patch could then be used by a more efficient fishery. The main issue is then to find which portion of the whole area should be opened as the second patch.

For doing that we considered the viewpoint of the regulators (in charge to fix the size of the patches). Using a calculus of variations approach we found conditions ensuring the existence of a reasonable cut maximizing the economic return.

The above presented results shows that it is possible to optimally distribute the growth area of the fishery between artisanal and industrial fisheries, given certain assumptions.

It is also observed that under certain conditions on biomass transfer it can be optimal to completely close the access of the industrial fishery to the growth area.

6 Appendix

We first present the results concerning the dynamical system for the biomass including harvesting.

Proposition 1. *Let the condition (5) be fulfilled and consider the dynamical system (4) for constant controls $\bar{E}_2$ and $\bar{\lambda}$ satisfying (6) and (7). There exist then two stationary points of (4). One of them is the zero vector. The nonzero one, $\bar{x}$, has positive components and is given by the equations (8) and (9). Furthermore, it holds that $\bar{x}$ is an stable node, but zero is not.*

Proof:

Let us denote $z = x_1 + x_2$. The stationary points of (4) can be equivalently defined as the solutions of the algebraic system

$$\gamma z(1 - \frac{1}{K}z) - q_1 E_1 x_1 - q_2 \bar{E}_2 (z - x_1) = 0 \quad (21)$$

$$\bar{\lambda}\gamma z(1 - \frac{1}{K}z) - (b + q_1 E_1) x_1 = 0 \quad (22)$$

Since by definition $(b+q_1E_1)>0$ the following relation must hold

$$x_1 = \frac{\bar{\lambda}\gamma}{(b+q_1E_1)} z(1-\frac{1}{K}z) \tag{23}$$

Using this in (21) leads to the following algebraic condition over z.

$$\gamma z(1-\frac{1}{K}z) - q_2E_2z = (q_1E_1 - q_2\bar{E}_2)\frac{\bar{\lambda}\gamma}{(b+q_1E_1)} z(1-\frac{1}{K}z)$$

There exists then the trivial solution $z=0$, which due to (23) leads to the first solution $x=0$. Taking into account the above algebraic condition over z it is clear that there exists another stationary point if and only if

$$\bar{\lambda}(q_1E_1 - q_2\bar{E}_2) \neq (b+q_1E_1). \tag{24}$$

In this case the other unique solution satisfies:

$$z = \frac{K}{\gamma}\frac{(\gamma - q_2\bar{E}_2)(b+q_1E_1) - \bar{\lambda}\gamma(q_1E_1 - q_2\bar{E}_2)}{(b+q_1E_1) - \bar{\lambda}(q_1E_1 - q_2\bar{E}_2)} \tag{25}$$

Using this value in (23) leads easily to (8). This also implies the relationschip

$$z = \frac{x_1}{\bar{\lambda}q_2\bar{E}_2}[(b+q_1E_1) - \bar{\lambda}(q_1E_1 - q_2\bar{E}_2)]$$

Now (9) holds trivially since $x_2 = z - x_1$.

The positivity of the nonzero solution requires that:

$$\bar{\lambda}(q_1E_1 - q_2\bar{E}_2) < \frac{(\gamma - q_2\bar{E}_2)}{\gamma}(b+q_1E_1) \tag{26}$$

which is equivalent to:

$$(\gamma - q_2\bar{E}_2)(b + (1-\bar{\lambda})q_1E_1) + q_2\bar{E}_2\bar{\lambda}(\gamma - q_1E_1) > 0$$

and it follows directly from the assumption of the proposition.

Finally let us point out that the condition (26) implies (24) since $(b+q_1E_1)$ and $\bar{E}_2$ are strictly positive. Consequently in this case there are actually two stationary points.

In order to prove the stability type of the two stationary points found let us write the system (4) in the following form

$$\begin{aligned} \dot{x}_1 &= \bar{\lambda}r(x_1,x_2) - (b+q_1E_1)x_1 \\ \dot{x}_2 &= (1-\bar{\lambda})r(x_1,x_2) + bx_1 - q_2\bar{E}_2x_2 \end{aligned}$$

where

$$r(x_1,x_2) = \gamma(x_1+x_2)(1-\frac{x_1+x_2}{K})$$

A stationary point $x = (x_1, x_2)$ is a stable one if and only if the matrix

$$C(x_1, x_2) = \left[\begin{array}{cc} \bar{\lambda}\frac{\partial r}{\partial x_1}(x_1, x_2) - (b + q_1 E_1) & \bar{\lambda}\frac{\partial r}{\partial x_2}(x_1, x_2) \\ (1-\bar{\lambda})\frac{\partial r}{\partial x_1}(x_1, x_2) + b & (1-\bar{\lambda})\frac{\partial r}{\partial x_2}(x_1, x_2) - q_2\bar{E}_2 \end{array} \right]$$

has negative trace and positive determinant. Taking into account that $\frac{\partial r}{\partial x_1}$ and $\frac{\partial r}{\partial x_2}$ coincide the stability is then equivalent to the conditions:

$$\frac{\partial r}{\partial x_1}(x_1, x_2) - (b + q_1 E_1 + q_2\bar{E}_2) < 0 \quad (27)$$

$$\bar{\lambda}(q_1 E_1 - q_2\bar{E}_2)\frac{\partial r}{\partial x_1}(x_1, x_2) + (b + q_1 E_1)[q_2\bar{E}_2 - \frac{\partial r}{\partial x_1}(x_1, x_2)] > 0 \quad (28)$$

From the definition of r it holds $\frac{\partial r}{\partial x_2}(0) = \gamma$ and therefore

$$det(C(0)) = \bar{\lambda}\gamma(q_1 E_1 - q_2\bar{E}_2) + (b + q_1 E_1)(q_2\bar{E}_2 - \gamma).$$

This is strictly negative due to (26). Consequently 0 is not a stable node.

The other stationary point $\bar{x} = (\bar{x}_1, \bar{x}_2)$ satisfies the relations (25) for $\bar{z} = \bar{x}_1 + \bar{x}_2$. Using that in the expression of the Trace

$$\bar{\alpha} = Trace(C(\bar{x})) = \gamma(1 - \frac{2}{K}\bar{z}) - (b + q_1 E_1 + q_2\bar{E}_2)$$

leads to:

$$\bar{\alpha} = \frac{\bar{B}\gamma - 2(\gamma - q_2\bar{E}_2)(b + q_1 E_1) - \bar{\lambda}\gamma(q_1 E_1 - q_2\bar{E}_2) - \bar{B}(b + q_1 E_1 + q_2\bar{E}_2)}{\bar{B}} \quad (29)$$

where $\bar{B} = (b + q_1 E_1) - \bar{\lambda}(q_1 E_1 - q_2\bar{E}_2)$. Due to (7) it holds

$$\bar{\lambda}(q_1 E_1 - q_2\bar{E}_2) < \frac{(\gamma - q_2\bar{E}_2)}{\gamma}(b + q_1 E_1) < (b + q_1 E_1) \quad (30)$$

and therefore $\bar{B} > 0$. Consequently the sign of $Trace(C(\bar{x}))$ only depends on the numerator of $\bar{\alpha}$ in (29). This numerator is equal to

$$-(b+q_1E_1)[(\gamma-q_2\bar{E}_2)+(b+q_1E_1)-\bar{\lambda}(q_1E_1-q_2\bar{E}_2)]+(\gamma+q_2\bar{E}_2)\bar{\lambda}(q_1E_1-q_2\bar{E}_2) \quad (31)$$

which due to (7), (30) and the feasibility of the controls is negative in the case $q_1E_1 - q_2\bar{E}_2 \leq 0$. It remains then only to prove that (31) is also negative when

$$q_1 E_1 - q_2\bar{E}_2 > 0 \quad (32)$$

Using the first inequality of (30) it can be easily seen that (31) is smaller than

$$-(b+q_1E_1)[(\gamma-q_2\bar{E}_2)+(b+q_1E_1)-\bar{\lambda}(q_1E_1-q_2\bar{E}_2)]+(\gamma+q_2\bar{E}_2)\frac{(\gamma - q_2\bar{E}_2)}{\gamma}(b+q_1E_1)$$

and therefore it suffices to show

$$-\gamma[(\gamma - q_2\bar{E}_2) + (b + q_1E_1) - \bar{\lambda}(q_1E_1 - q_2\bar{E}_2)] + \gamma^2 - (q_2\bar{E}_2)^2 < 0$$

This follows but inmediatly from (32) and the feasibility of $\bar{\lambda}$ since the left hand side redudes to $-(q_2\bar{E}_2)^2 - \gamma[b + (1-\bar{\lambda})(q_1E_1 - q_2\bar{E}_2)]$.

Finally we have to prove that (28) holds true at $\bar{x}$. Note that

$$\bar{\beta} = Det(C(\bar{x})) = \bar{\lambda}\gamma(1 - \frac{2}{K}\bar{z})(q_1E_1 - q_2\bar{E}_2) + (b + q_1E_1)[q_2\bar{E}_2 - \gamma(1 - \frac{2}{K}\bar{z})],$$

or equivalently

$$\bar{\beta} = q_2\bar{E}_2(b + q_1E_1) - \gamma(1 - \frac{2}{K}\bar{z})[(b + q_1E_1) - \bar{\lambda}(q_1E_1 - q_2\bar{E}_2)].$$

Using (25) for $\bar{z}$ gives then

$$\bar{\beta} = (\gamma - q_2\bar{E}_2)(b + q_1E_1) - \bar{\lambda}\gamma(q_1E_1 - q_2\bar{E}_2).$$

This is but strictly positive due to (30). ■

The next proposition deals with the optimality condition to the underlying calculus of variation problem.

Proposition 2. *First order optimal conditions of the problem (11) are given by the following system corresponding to the Euler-Lagrange equations.*

$$\begin{array}{rcl} \dot{x}_1 &=& x_2(\frac{pq_2}{c_2}x_2 - 1)[\gamma(1 - 2\frac{x_1+x_2}{K}) - \delta] + \gamma(x_1+x_2)(1 - \frac{x_1+x_2}{K}) - q_1E_1x_1 \\ \dot{x}_2 &=& -x_2(\frac{pq_2}{c_2}x_2 - 1)[\gamma(1 - 2\frac{x_1+x_2}{K}) - \delta] \end{array} \quad (33)$$

Proof: Consider the Euler-Lagrange optimality condition:

$$l_x - \frac{d}{dt}l_{\dot{x}} + \delta l_{\dot{x}} = 0$$

First we need the partial derivatives of $l(x, \dot{x})$ with respect to x:

$$\begin{array}{rcl} l_{x_1} &=& (p - \frac{c_2}{q_2x_2})[\gamma(1 - 2\frac{x_1+x_2}{K}) - q_1E_1] + pq_1E_1 \\ l_{x_2} &=& \frac{c_2}{q_2x_2^2}[\gamma(x_1+x_2)(1 - \frac{x_1+x_2}{K}) - q_1E_1x_1 - \dot{x}_1 - \dot{x}_2] + (p - \frac{c_2}{q_2x_2})\gamma(1 - 2\frac{x_1+x_2}{K}) \end{array}$$

and with respect to $\dot{x}$:

$$\begin{array}{rcl} l_{\dot{x}_1} &=& -(p - \frac{c_2}{q_2x_2}) \\ l_{\dot{x}_2} &=& l_{\dot{x}_1} \end{array}$$

Therefore

$$\frac{d}{dt}l_{\dot{x}_1} = \frac{d}{dt}l_{\dot{x}_2} = -\frac{c_2\dot{x}_2}{q_2x_2^2}$$

Stating the Euler-Lagrange equation for x_1 we obtain:

$$(p - \frac{c_2}{q_2 x_2})[\gamma(1 - 2\frac{x_1 + x_2}{K}) - q_1 E_1] + pq_1 E_1 + \frac{c_2 \dot{x}_2}{q_2 x_2^2} - \delta(p - \frac{c_2}{q_2 x_2}) = 0$$

or equivalently

$$(p - \frac{c_2}{q_2 x_2})[\gamma(1 - 2\frac{x_1 + x_2}{K}) - q_1 E_1 - \delta] + pq_1 E_1 = -\frac{c_2 \dot{x}_2}{q_2 x_2^2}$$

This leads to

$$-\dot{x}_2 = x_2(\frac{pq_2}{c_2} x_2 - 1)[\gamma(1 - 2\frac{x_1 + x_2}{K}) - q_1 E_1 - \delta] + \frac{pq_1 q_2 E_1}{c_2} x_2^2$$

and finally

$$\dot{x}_2 = -x_2(\frac{pq_2}{c_2} x_2 - 1)[\gamma(1 - 2\frac{x_1 + x_2}{K}) - \delta]$$

The Euler-Lagrange Equation for x_2 gives:

$$\begin{aligned} 0 \quad = \quad & \frac{c_2}{q_2 x_2^2}[\gamma(x_1 + x_2)(1 - \frac{x_1 + x_2}{K}) - q_1 E_1 x_1 - \dot{x}_1 - \dot{x}_2] + \\ & +(p - \frac{c_2}{q_2 x_2})\gamma(1 - 2\frac{x_1 + x_2}{K}) + \frac{c_2 \dot{x}_2}{q_2 x_2^2} - \delta(p - \frac{c_2}{q_2 x_2}) \end{aligned}$$

which transforms into

$$0 = \gamma(x_1 + x_2)(1 - \frac{x_1 + x_2}{K}) - q_1 E_1 x_1 - \dot{x}_1 + x_2(\frac{pq_2}{c_2} x_2 - 1)[\gamma(1 - 2\frac{x_1 + x_2}{K}) - \delta]$$

and finally

$$\dot{x}_1 = x_2(\frac{pq_2}{c_2} x_2 - 1)[\gamma(1 - 2\frac{x_1 + x_2}{K}) - \delta] + \gamma(x_1 + x_2)(1 - \frac{x_1 + x_2}{K}) - q_1 E_1 x_1 \quad \blacksquare$$

Let us now study the existence of strictly positive stationary points of the Euler-Lagrange system (12).

Proposition 3. *If the condition (5) is satisfied and any of the two conditions (13) or (14 - 15) holds true, then there exists exactly one stationary point* $x^* = (x_1^*, x_2^*)$ *of (12) such that* $x_1^* > 0$ *and* $x_2^* > 0$*. These levels are given by (16) and (17).*

Proof:

Let us first note that from any of the both sufficient conditions proposed follows easily the existence of some $\alpha > 0$ satisfying:

$$q_1 E_1 < \frac{\gamma + \delta}{(2 + \alpha)} \tag{34}$$

$$\frac{pq_2 K}{c_2} > \left(1 + \frac{1}{\alpha}\right)\left(\frac{\gamma}{\gamma - q_1 E_1}\right) \tag{35}$$

To localize the stationary points means nothing else but solving the following algebraic equations.

$$\begin{aligned} 0 &= x_2(\frac{pq_2}{c_2}x_2 - 1)[\gamma(1 - 2\frac{x_1 + x_2}{K}) - \delta] + \gamma(x_1 + x_2)(1 - \frac{x_1 + x_2}{K}) - q_1E_1x_1 \\ 0 &= -x_2(\frac{pq_2}{c_2}x_2 - 1)[\gamma(1 - 2\frac{x_1 + x_2}{K}) - \delta] \end{aligned}$$

Adding up the equations deal to the condition

$$0 = \gamma(x_1 + x_2)(1 - \frac{x_1 + x_2}{K}) - q_1E_1x_1$$

or equivalently

$$(x_1 + x_2)[(\gamma - q_1E_1) - \frac{\gamma}{K}(x_1 + x_2)] = 0$$

From the above equation follows that $x_1 + x_2$ equals zero or $\frac{K}{\gamma}(\gamma - q_1E_1)$. We are interested only in positive solutions and therefore must take

$$x_1 + x_2 = \frac{K}{\gamma}(\gamma - q_1E_1). \tag{36}$$

Since we are interested only on positive solutions, it is sufficient to consider the case in which:

$$\gamma - \delta - \frac{2\gamma}{K}(x_1 + x_2) = -\frac{q_1E_1}{(\frac{pq_2}{c_2}x_2 - 1)}. \tag{37}$$

Since the expression obtained for $x_1 + x_2$ does not depend on x_2, equation (37) becomes linear in x_2 giving rise to:

$$x_2^* = \frac{c_2(\gamma + \delta - q_1E_1)}{pq_2(\gamma + \delta - 2q_1E_1)}$$

and according to (36)

$$x_1^* = \frac{K}{\gamma}(\gamma - q_1E_1) - \frac{c_2(\gamma + \delta - q_1E_1)}{pq_2(\gamma + \delta - 2q_1E_1)}$$

The assumption (34) implies that

$$\gamma + \delta - 2q_1E_1 > \alpha q_1E_1 > 0 \tag{38}$$

and consequently $x_2^* > 0$.

Taking into account the hypothesis (5) the condition (35) can be stated as

$$\frac{K}{\gamma}(\gamma - q_1E_1) > \frac{c_2}{pq_2}\left(1 + \frac{1}{\alpha}\right)$$

But from (38) it follows immediately

$$\frac{1}{\alpha} > \frac{q_1E_1}{\gamma + \delta - 2q_1E_1}$$

Combining the last two inequalities leads to

$$\frac{K}{\gamma}(\gamma - q_1E_1) > \frac{c_2}{p_2q_2}\left(1 + \frac{q_1E_1}{\gamma+\delta-2q_1E_1}\right) = \frac{c_2}{pq_2}\frac{(\gamma+\delta-q_1E_1)}{(\gamma+\delta-2q_1E_1)}$$

This is finally equivalent to $x_1^* > 0$ and we are done. ■

Proposition 4. *The values of the two controls associated to the steady states x_1^*, x_2^* are positive and given by the expressions (18) and (19).*

Proof: Let $x^* = (x_1^*, x_2^*)$ be the unique positive stationary point of the Euler-Lagrange system of equation (12) described in the above section. If we want this point to be a stationary solution of the original system (4) using constant controls $E_2^*(t) = E_2^*, \lambda^*(t) = \lambda^*$, then the constants must satisfy the following relations.

$$E_2^* = \frac{1}{q_2x_2^*}\left[\gamma z^*\left(1-\frac{1}{K}z^*\right) - q_1E_1x_1^*\right] \tag{39}$$

$$\lambda^* = \frac{bx_1^* + q_1E_1x_1^*}{\gamma z^*\left(1-\frac{1}{K}z^*\right)} \tag{40}$$

Here z^* denotes the sum of both componetns (i.e. $z^* = x_1^* + x_2^*$). These are then the constant controls associated with the unique feasible stationary point of the Euler-Lagrange equation. If the above values are feasible we can construct an interior solutions for the controls satisfying first order optimality.

If we define $B^* = (\gamma - q_1E_1)^2$, then z^* can be also written as

$$z^* = \frac{K}{2\gamma}[(\gamma - q_1E_1) + \sqrt{B^*}].$$

Using that in (39) leads to

$$\begin{aligned} E_2^* &= \frac{1}{q_2x_2^*}\left[\frac{K}{2}[(\gamma-q_1E_1)+\sqrt{B^*}]\left(1-\frac{1}{2\gamma}[(\gamma-q_1E_1)+\sqrt{B^*}]\right) - q_1E_1x_1^*\right] \\ &= \frac{1}{q_2x_2^*}\left[\frac{K}{4\gamma}[\gamma-q_1E_1+\sqrt{B^*}]\left(\gamma-q_1E_1-\sqrt{B^*}+2q_1E_1\right) - q_1E_1x_1^*\right] \\ &= \frac{1}{q_2x_2^*}\left[\frac{K}{4\gamma}[(\gamma-q_1E_1)^2-B^*] + q_1E_1z^* - q_1E_1x_1^*\right]. \end{aligned}$$

Now from the definition of B^* and $z^* = x_1^* + x_2^*$ it follows

$$E_2^* = \frac{q_1E_1}{q_2}$$

The expression (19) follows straightforwardly from (40) and (18) taking into account that $\gamma z^*\left(1-\frac{1}{K}z^*\right) = q_1E_1x_1^* + q_2E_2^*x_2^*$. ■

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